Simulation of angioplasty using isogeometric laminated composite shell elements

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Abstract: The typical treatment for the opening of obstructed arteries or veins is balloon angioplasty. Different computational models, which usually use 3D finite solid elements to model the artery, have been introduced in the literature to analyze the angioplasty procedure. In this work, a laminated composite shell formulation derived from the Kirchhoff–Love shell theory is used to model the artery wall. The corresponding shell is formulated on the mid-surface of the artery. As only the surface is discretized, the shell formulation is computationally less expensive than the common 3D solid elements, which require volumetric discretization.

1 Introduction

Atherosclerosis, a disease in which plaque builds up inside arteries or veins, is typically treated by balloon angioplasty. Several computational studies based on finite element analysis (e.g. Gasser and Holzapfel (2007)) have been performed to optimize the angioplasty parameters, among which the most important ones are the internal pressure, location and mechanical properties of the balloon. Most numerical simulations of angioplasty use 3D solid elements to model artery walls. However, as shown in the presented work, shell elements, which are computationally less expensive, are sufficiently accurate to describe and analyze the walls of arteries and veins in many circumstances.

Here, the isogeometric shell formulation of Roohbakhshan and Sauer (2016, 2017) and Roohbakhshan et al. (2017) is used to model an artery. The formulation allows for the modeling of laminated composite shells constructed from different materials, like the adventitia, media and intima layers of an artery. The laminated composite shell formulation is based on the Kirchhoff–Love hypothesis and both the artery and balloon are discretized by NURBS-based finite elements. Furthermore, the numerical results are compared with those obtained by using 3D solid elements. The comparison shows very good agreement. For the sake of brevity, the key concepts of the laminated composite shell theory, i.e. the kinematics, weak form of the boundary value problem, finite element formulation and constitutive modeling are skipped here. The reader is recommended to see Duong et al. (2017) and Roohbakhshan and Sauer (2016, 2017) for more details.

2 Artery constitution

As shown in Fig. 1.a, the artery is constructed from three layers, namely intima, media and adventitia. The matrial behavior of all three layers is modeled by the anisotropic hyperelastic

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material model of Gasser et al. (2006). For each layer, different thickness T_l and material properties are assumed as summarized in Tab. 1. Here, index $l = \mathrm{I}, \mathrm{M}, \mathrm{A}$ stands for the intima, media and adventitia layers, respectively.

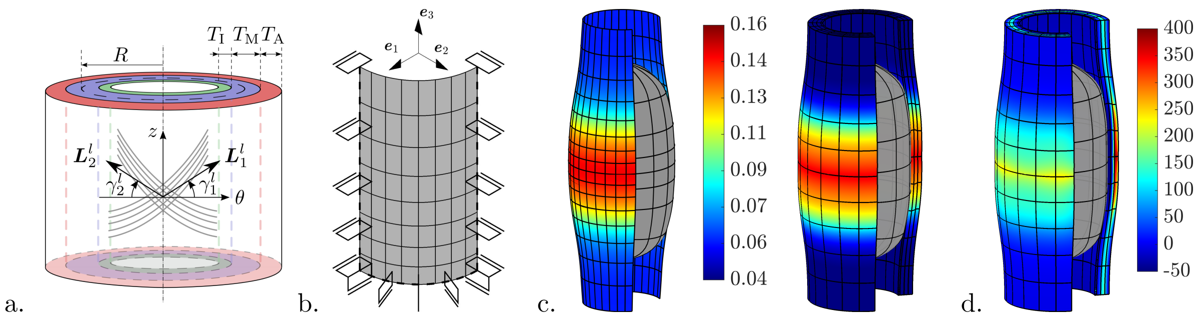

Figure 1: Artery model and deformed configuration: a. Artery three-layer model. b. Computational model with boundary conditions. c. Deformed artery, modeled by the finite shell (left) and solid (right) elements, colored by the first invariant of the 2D projected stress tensor $I_\sigma = \mathrm{tr}\,\boldsymbol{\sigma}$ [kNm^{-1}]. d. Deformed artery, modeled by the finite solid elements, colored by the first invariant of the 3D stress tensor $I_{\tilde{\sigma}} = \mathrm{tr}\,\tilde{\boldsymbol{\sigma}}$ [kNm^{-2}].

Layer	T_l [μm]	$\tilde{\mu}_l$ [kPa]	$\tilde{k}^l_{1i}$ [kPa]	k^l_{2i}	γ^l_i [deg]	κ^l_i
Intima	138	28	2.0×10^3	1000	± 40	0.052
Media	787	1.3	0.5×10^3	50	± 30	0.046
Adventitia	428	7.5	1.0×10^3	500	± 50	0.055

Table 1: Material properties of the three layers of the artery (Roohbakhshan et al., 2017).

The 3D strain-energy density function $\tilde{W}^l = \tilde{W}^l_\mathrm{m} + \tilde{W}^l_\mathrm{f}$, proposed by Gasser et al. (2006), has an isotropic part $\tilde{W}^l_\mathrm{m}$, due to the ground matrix, and an anisotropic part, $\tilde{W}^l_\mathrm{f}$, due to two families of collagen fibers. The isotropic part is modeled by a Neo–Hookean solid, which needs the shear modulus $\tilde{\mu}$ as the material constant. The anisotropic part is given by

$$\tilde{W}^l_\mathrm{f} := \sum_{i=1}^{2} \frac{\tilde{k}^l_{1i}}{2\,k^l_{2i}} \left\{ \exp\left[k^l_{2i}\,(\tilde{J}^l_{4i} - 1)^2 \right] - 1 \right\} , \tag{1}$$

where $\tilde{J}^l_{4i}$ are the invariants of the 3D generalized structural tensor, introduced by Gasser et al. (2006), which depend on the dispersion parameter κ^l_i and the principal direction of fibers $\boldsymbol{L}^l_i$. Following Figs. 1a and b, $\boldsymbol{L}^l_i = \cos\gamma^l_i\,\sin\psi\,\boldsymbol{e}_1 + \cos\gamma^l_i\,\cos\psi\,\boldsymbol{e}_2 + \sin\gamma^l_i\,\boldsymbol{e}_3$ $(i = 1, 2)$, where γ^l_i is given in Tab. 1 and ψ is the angular coordinate around the $\boldsymbol{e}_3$ axis.

3 Angioplasty simulation

As shown in Fig. 1, an artery is modeled as a $T \times R \times L = 1.35 \times 5 \times 30$ [mm^3] cylinder inflated by a balloon. Due to the symmetry of the problem, only 1/8 of the artery and balloon are modeled and the symmetry constraints are imposed following Duong et al. (2017). At the ends of the artery, the axial displacement is set to zero as a Dirichlet boundary condition. The contact constraint is enforced following an unbiased penalty formulation (Sauer and De Lorenzis, 2015).

In Roohbakhshan et al. (2017), only the stress distribution across the shell layers was shown. Here, the solid model is also plotted for comparison. Fig. 1c shows the deformed artery modeled by finite shell and solid elements. On the left, the artery – discretized by the shell elements – is colored by the first invariant of the 2D projected stress tensor $I_\sigma = \mathrm{tr}\,\boldsymbol{\sigma}$. On the right, the artery – modeled by the finite solid elements – is colored by the first invariant of the equivalent projected stress, which is obtained by integrating the 3D stress tensor $\tilde{\boldsymbol{\sigma}}$ through the thickness of the artery wall (see Fig. 1d). As shown in Fig. 1d, the adventitia layer contributes most to the artery strength. Fig. 2 compares the radial displacement and circumferential stretch of both shell and solid models. Although the artery is rather thick to be modeled by laminated composite Kirchhoff–Love shells, the results are in good agreement with those obtained by finite solid elements. The minor differences are due to the fact that, in the shell model, the contact constraint is enforced on the mid-surface of the artery while it is enforced on the inner surface for the solid model.

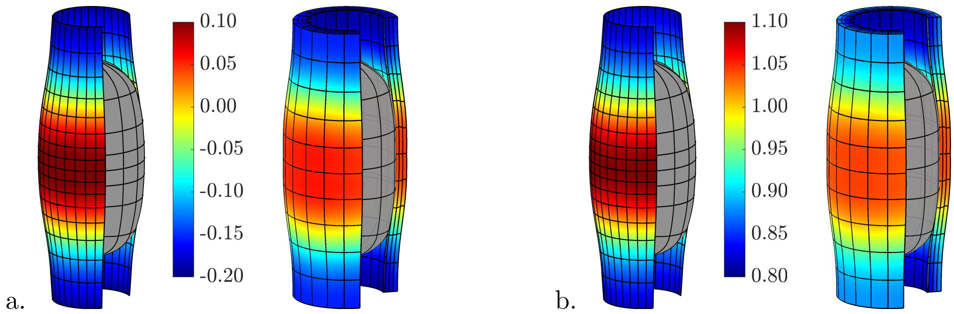

Figure 2: Deformed artery, modeled by the finite shell (left) and solid (right) elements, colored by the radial displacement (a. normalized by R) and circumferential stretch (b.).

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