Master of Science "Computational Engineering", Summer 2003

Course

ADVANCED FINITE ELEMENT METHODS

Written Examination on 06.10.2003

Last Name: ____________________ First name: ____________________ Matr.-No.: __________
(please write legibly)

problem	1	2	3	4	sum
possible points	40	30	45	65	180
obtained points					

Important instructions

- Duration: 3 hours,
  first 40 minutes without appliance,
  2 hours and 20 minutes with appliance.
- Prove the completeness of the problems.
- Write your name onto all pages of the test.
- Write the solutions of problem no. 1 onto the coloured handout. Don't use your own paper.
- Hand in all pages of the examination.
- Don't use green coloured pens.
- The solutions should include all auxiliary calculations.
- Programmable pocket calculators are only allowed without programs.
- The use of laptops or notebooks is not allowed.
- Individual visit of the toilet is permitted.
- Don't leave the lecture room between part I and part II of the examination and stay in the room until the preparation time is over.
- Please deactivate your mobile phone.

Problem 1 max. $\sum$ points: 40 obtained $\sum$ points:

Problem 1.1 (max. points: 5)

Quote two different types of non-linearity in structural mechanics. Which fundamental equations of linear structural mechanics have to be changed? Specify these equations in general form.

Problem 1.2 (max. points: 4)

Sketch the load-displacement diagram for a one-dimensional loading-unloading cycle with

a) elasto-plastic material behaviour.

b) elasto-damaging material behaviour.

Problem 1.3 (max. points: 4)

Perform the Gâteaux derivative of

$$f(x) = \mathrm{e}^{\frac{1}{x}} \tag{1}$$

with respect to x.

Problem 1.4 (max. points: 2)

Quote two possible control methods that are able to compute equilibrium points beyond the maximum of a load-displacement curve.

Problem 1.5 (max. points: 4)

Sketch a load-displacement diagram of a structure with

a) stable behaviour.

b) possible unstable behaviour.

Explain why you expect stable / unstable behaviour.

Problem 1.6 **(max. points: 4)**

Which types of critical points have to be distinguished in structural mechanics? Draw a load-displacement curve containing these critical points. Specify one possible condition for a critical point.

Problem 1.7 **(max. points: 5)**

In an arc-length controlled solution algorithm the equilibrium point has to satisfy an additional constraint $f(\boldsymbol{u}_{n+1}^{k+1}, \lambda_{n+1}^{k+1}) = 0$. Show that the corresponding load-increment can be computed as follows:

$$\Delta\lambda = -\frac{f(\boldsymbol{u}_{n+1}^{k}, \lambda_{n+1}^{k}) + f_{,u}(\boldsymbol{u}_{n+1}^{k}, \lambda_{n+1}^{k}) \cdot \Delta\boldsymbol{u}_r}{f_{,u}(\boldsymbol{u}_{n+1}^{k}, \lambda_{n+1}^{k}) \cdot \Delta\boldsymbol{u}_\lambda + f_{,\lambda}(\boldsymbol{u}_{n+1}^{k}, \lambda_{n+1}^{k})} \,. \tag{2}$$

Problem 1.8 **(max. points: 4)**

One possible constraint for an arc-length controlled solution algorithm is given in the following equation (initial normal plane):

$$f(\boldsymbol{u}_{n+1}^{k+1}, \lambda_{n+1}^{k+1}) = [\boldsymbol{u}_{n+1}^{1} - \boldsymbol{u}_n] \cdot [\boldsymbol{u}_{n+1}^{k+1} - \boldsymbol{u}_{n+1}^{k}] + [\lambda_{n+1}^{1} - \lambda_n]\,[\lambda_{n+1}^{k+1} - \lambda_{n+1}^{k}] \,. \tag{3}$$

Specify the corresponding load-increment $\Delta\lambda$ using equation (2).

Problem 1.9 **(max. points: 3)**

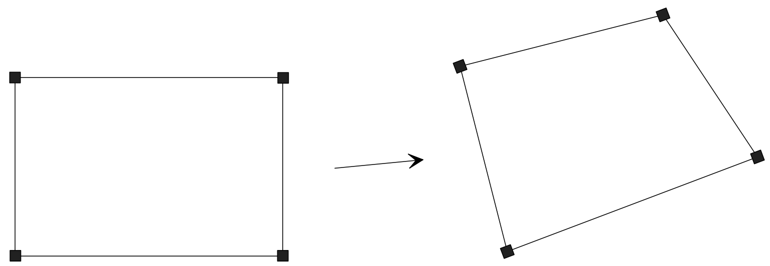
The principle of virtual work in geometrical non-linear theory contains a stress- and a strain tensor. Which tensors can be used for example? Can any strain tensor be combined with any stress tensor?

Problem 1.10 **(max. points: 5)**

Quote three different iteration schemes for the solution of the equilibrium equations in non-linear structural mechanics and describe their main characteristics.

Problem 2

max. $\sum$ points: 30 obtained $\sum$ points:



reference configuration current configuration

Node	reference configuration	current configuration
1	$(0\,/\,0)$	$(\frac{3}{2}\,/\,\sqrt{3})$
2	$(2\,/\,0)$	$(\frac{3}{2}+\sqrt{3}\,/\,\sqrt{3}+1)$
3	$(2\,/\,1)$	$(1+\sqrt{3}\,/\,1+\frac{3\sqrt{3}}{2})$
4	$(0\,/\,1)$	$(1\,/\,\frac{3\sqrt{3}}{2})$

The geometrically non-linear analysis of a slab has resulted in the deformed shape described by the nodal coordinates given in the table. The computation has been carried out by means of a four node finite element with bilinear displacement interpolations.

a) Compute the displacement field $\boldsymbol{u}(\boldsymbol{X})$ with respect to physical coordinates.

b) Determine the gradient of the displacement field $\nabla\boldsymbol{u}$.

c) Specify the deformation gradient $\boldsymbol{F}$.

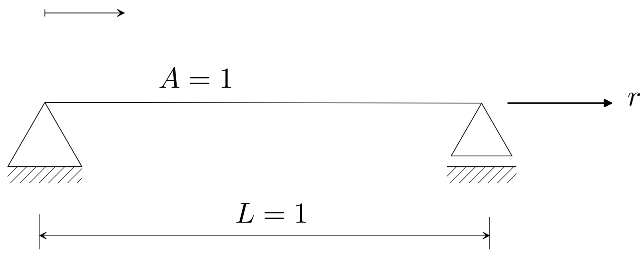
d) Calculate the components of the Green-Lagrange strain tensor and comment your results.

Problem 3 max. $\sum$ points: 45 obtained $\sum$ points:

The physically non-linear material response of an one dimensional truss element can be approximated by the free energy function

$$\psi(\varepsilon_{11}) = -\ln(\varepsilon_{11} + 1) + \exp(\varepsilon_{11}) - 1\,, \tag{4}$$

which depends on the linearized strain tensor ε_{11}. The application of the proposed material model should be illustrated by means of the following system:



a) Sketch the given free energy function $\psi(\varepsilon_{11})$ and the resulting constitutive law $\sigma_{11}(\varepsilon_{11}) = \dfrac{\partial\psi(\varepsilon_{11})}{\partial\varepsilon_{11}}$.

b) Enumerate the most important characteristics of the chosen model.

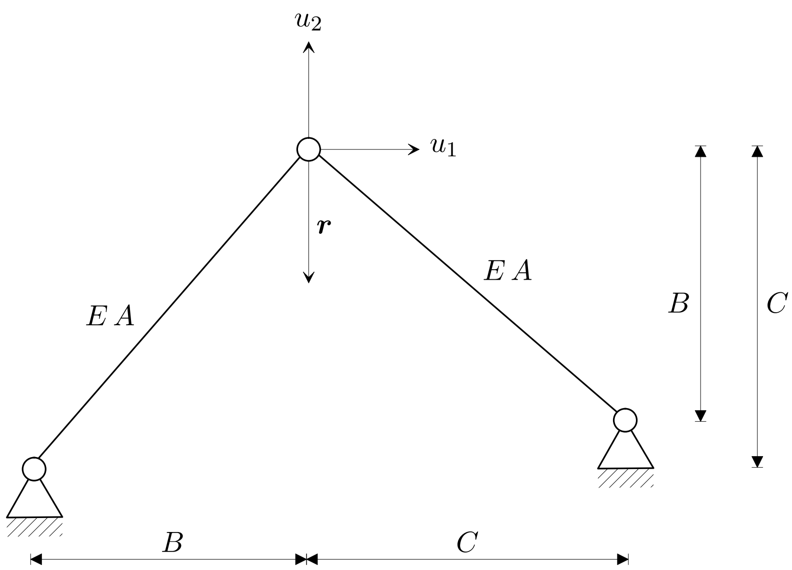
c) Compute the internal force vector r_i. The discretization should be carried out by means of one Finite Element with linear displacement interpolations.

d) Derive the tangential stiffness matrix K_t.

e) Compute the nodal displacement of the shown system for an external load $r = 7.05572$. The non-linear equation has to be solved by using the Newton-Raphson method with three iteration steps an one load-control step with $\lambda = 1$.

f) Choose a convergence check and prove the convergence.

Problem 4 max. $\sum$ points: 65 obtained $\sum$ points:



a) Sketch the equilibrium states that correspond to the load factor $\lambda = 0$. Do these results give any information whether the considered structure is susceptible to loss of stability?

b) Specify the vector of internal forces by direct generation of static equilibrium in the deformed state.

c) Show that the application of the CRISFIELD truss element leads to the same result.

d) Specify the tangential stiffness matrix.

For $B = C$ and vertical load, the system reduces to a one DOF-system. Use this simplified system for the following parts of this problem $\big($e) to i)$\big)$.

e) Specify the internal force vector and the tangential stiffness matrix for the simplified system.

f) Calculate the critical point u_{crit}, defined by the local maximum of the internal force r_i, in an analytical manner.

g) Sketch the equilibrium curve and mark the critical point.

h) Generate the extended system for the numerical calculation of the critical load factor λ_{crit}.

i) Linearize the related equations and give the matrix form of the linearized extended system. Specify the required derivatives, but **do not solve the system**.